

**Fig.1.**: Müller matrix element  $M_{32}$  for EPR ellipsometry scans of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> at 155GHz at different azimuthal sample orientation. The (-201) oriented sample was measured at 45° angle of incidence with the magnetic field B parallel to the incident light. The sample was rotated about its surface normal starting from B being parallel to the monoclinic plane close to the *c*\* direction of the crystal (0°). The signature of Fe<sup>3+</sup> (s=5/2) incorporated at different lattice sites is recognized as two quintuplets in each scan.

$$H = \mu_B \vec{B}g\vec{s} + \sum_{\substack{k=2\\k \text{ even } q \text{ even}}}^4 \sum_{\substack{q=-k\\q \text{ even }}}^k B_k^q O_k^q$$

Eq.1: Spin Hamiltonian for a monoclinic s=5/2 electronic system (without nuclear spin) with highsymmetry direction parallel to z in terms of Stevens (equivalent) operators  $O_k^q$  and coefficients  $B_k^q$ . The first term is the normal Zeeman splitting with Bohr magneton  $\mu_B$ , magnetic field  $\vec{B}$ , g-factor tensor g, and spin  $\vec{s}$ . Terms with k = 4 would vanish for s=3/2, terms with negative indices q would vanish for orthorhombic systems under appropriate choice of the coordinate system. For a truly monoclinic s=3/2 systems,  $B_2^{-2}$  would be zero in a coordinate system where g would not be diagonal. For Fe<sup>3+</sup> in Ga<sub>2</sub>O<sub>3</sub> (Fig.1),  $B_4^{\pm 2,4}$  turn out to be nonnegligible.